Municipality-level estimates of child mortality for Brazil:
A new approach using Bayesian Statistics

Sarah McKinnon ¹
Joseph E. Potter¹
Carl S. Schmertmann²

¹Population Research Center, University of Texas at Austin
²Center for Demography & Population Health, Florida State University
Abstract

Previous efforts to estimate child mortality levels in small geographical areas have been hampered by the relative rarity of child deaths, which has often resulted in unstable and unreliable estimates. However, with a spatial smoothing process based upon Bayesian Statistics it is possible to “borrow” information from neighboring areas in order to generate more stable and accurate estimates of mortality in smaller areas. The objective of this study is to use this spatial smoothing process to derive estimates of child mortality at the level of the municipality in Brazil. Using data from the 2000 Brazil Census, I derive both Bayesian and non-Bayesian estimates of mortality for each municipality. In comparing the smoothed and raw estimates of this parameter, I find that the Bayesian estimates yield a clearer spatial pattern of child mortality with smaller variances in less populated municipalities, thus, more accurately reflecting the true mortality situation of those municipalities.
Introduction

In Brazil, child mortality has declined substantially in the last several decades from 177 deaths per 1,000 live births in 1960 to only 33 deaths per 1,000 live births in 2005 (UNICEF 2006). Yet, rates of mortality decline have slowed in recent years. While rates declined, on average, by 3.9 deaths per 1,000 live births per year between 1960 and 1990, the decline in rates were much lower (1.8 deaths per 1,000 live births) between 1990 and 2005. In addition, when compared to other countries, Brazil’s child mortality rates remain elevated. According to the World Health Organization (2006), child mortality rates in Brazil are higher than those of the United States and Canada as well as a number of other Latin American countries including Argentina, Columbia, Ecuador, Mexico, Panama, and Venezuela. In fact, Brazil’s child mortality rates are most similar to those of the Dominican Republic, Belize, Nicaragua, and Honduras, countries whose combined Gross Domestic Product is substantially below that of Brazil (World Bank 2007).

Additionally, child mortality rates in Brazil have been shown to vary considerably throughout the country. There are strong regional differentials in mortality rates with the North and Northeast consistently demonstrating much higher rates than the South and Southeast regions (e.g. Carvalho 1974, Alves 2003). And, while regional differentials in levels and declines in child mortality Brazil are both large and persistent, there is also evidence of even greater differentials at smaller geographic levels. For example, Alves (2003) found that infant mortality rates in the year 2000 ranged from a low of 8.5 deaths per 1,000 births in municipalities in the South to rates as high as 110 deaths per 1,000
live births in municipalities in the North – rates that are similar to those found in poor African countries (United Nations Development Programme 2003).

The Task Force on Child Health and Maternal Health, established by the United Nations to monitor and evaluate progress towards improvements in maternal and child health, concluded that “deep inequities in health status and access to healthcare both between and, equally important, within countries” (Freedman et al. 2005 p. xi) are a major contributor to the difficulties experienced by countries trying to reduce child mortality. According to the task force, efforts to improve maternal and child health must shift from a “one-size-fits-all” approach to one that includes “the intimate spaces of families, households, and communities” (p.2). They recommend that initiatives for assessing and addressing health status and access to care should have a much smaller geographical focus, stating that “until initiatives genuinely draw on context-specific knowledge and local capacity, health initiatives will not succeed at scale” (p.22).

Consequently, those working towards further reductions child mortality in countries such as Brazil have begun to focus on identifying levels of child mortality for smaller geographical areas. In Brazil, the United Nations Development Programme (UNDP) now publishes child mortality rates for each municipality in Brazil. However, these rates are constructed using birth and death certificate data (UNDP 2009) which have some serious limitation. For example, a study of the vital registration system in Brazil from 2003 to 2005 found that information on births was inadequate for 13% of the total population and as high as 28% for the population in the poorest region of the country, the Northeast (Szwarcwald 2008). Figure 1 presents municipal-level child mortality rates (per 1,000 live births) published by the UNDP for the year 2000. Of the
5,505 municipalities shown, 738 are lacking sufficient data for the construction of child mortality rates. Additionally, although it is well-established that child mortality rates are higher in the northern areas of the country and lower in the southern areas, there is very little indication of this in the map. The pattern in municipal-level mortality rates is sporadic with a great amount of variation in the rates for all regions of the country.

Constructing rates for relatively rare events such as child mortality for small geographical areas can often be unstable as just a few more or a few less child deaths can greatly impact the estimates in areas with small populations. In Brazil, approximately one-quarter of all municipalities have less than 5,000 residents while more than half have less than 10,000 people. And, with an average child mortality rate of 33 deaths per 1,000 births in Brazil, it is likely that in many of the municipalities it will be difficult to create accurate estimates of child mortality. This idea is illustrated in Figure 2, which plots the UNDP municipal-level child mortality rates by the population of the municipalities. While child mortality rates are fairly stable and consistent for the municipalities with the larger populations, there is a lot of variation in the rates for the municipalities with the smaller populations. The high level of instability in child mortality rates in areas with small populations is even more evident when comparing Figure 3 which plots the rates for the municipalities with the 10% smallest population sizes with Figure 4 which plots the rates for the municipalities with the 10% smallest population sizes. Whereas the rates for Figure 3 are widely scattered above and below the mean value of 41.5 deaths per 1,000 live births, in Figure 4 they are all fairly concentrated around the mean of 24.6.

1 Due to scaling issues, 6 municipalities with the largest population sizes are omitted from the figures. All statistical measures include information on these municipalities.
Table 1 displays the measures of dispersion for the municipal level child mortality rates provided by the UNDP for all municipalities, municipalities with the 10% smallest populations, and municipalities with the 10% largest populations. Once again, it is very clear that child mortality rates for small populations are very unstable. Not only is the range in child mortality rates far greater for the least populated municipalities but also the variance is more than eight times higher and the standard deviation is almost three times higher in the least populated municipalities compared with the most populated municipalities.

In Brazil, child mortality is highly associated with female education, both at the level of the individual (e.g. Merrick 1985; Sastry 2004) and the community (e.g. Goldani et al. 2002; Alves 2003). As a result, if municipal-level estimates of child mortality are accurate, areas with lower levels of education should exhibit higher $q(5)$ values while areas with higher levels of education should exhibit lower values. To explore this issue, Table 1 provides the correlation coefficient for the relationship between years of schooling of women aged 25 and above and the child mortality rate. While all municipalities and the most populated municipalities exhibit a significant negative relationship between education and child mortality, there is a very weak and nonsignificant relationship for the least populated municipalities, indicating that the child mortality rates for these areas may be unreliable.

Thus, while efforts have been made to identify levels of child mortality for small geographical areas in Brazil, according to UN recommendations, it is clear that the rates have some significant weaknesses and, especially in areas with small populations, cannot be used with much confidence. Accordingly, the objective of this study is to create more
stable and reliable estimates of child mortality for each of the 5,505 municipalities in Brazil. Using a spatial smoothing process based upon Bayesian Statistics, we will be able to “borrow” information from neighboring areas in order to generate more stable estimates of mortality in smaller areas, providing valuable information on the true mortality risks of children residing throughout all areas of Brazil.

**Data and Methods**

The primary data used for this study is 2000 Brazil Census microdata collected from a 20% sample of households located in municipios with less than 15,000 residents and a 10% sample of households in municipios with 15,000 or more residents. The microdata is derived from a long-form questionnaire which includes questions on household conditions and amenities, income and occupation status, literacy and education, race, religion, marital status, migration, and parity. The main unit of analysis of this study is the municipality (n=5,505) which is determined by a geographic identifier included with the Census data (Figure 5). Municipal-level measures of female education are constructed by summing the total number of years of schooling for every woman aged 25 and above in each municipality. The sums are then divided by the total number of women aged 25 and above to calculate the mean number of years of schooling for women aged 25 and above in each municipality.

While the long form questionnaire does ask each woman how many children they had and the number of children still alive, it does not include sufficient detailed information on their reproductive histories to allow for the direct calculation of child mortality rates. Yet, it is possible to use Census information that is available to create indirect estimates of child mortality. Indirect estimates of child mortality, first introduced
by William Brass in 1968 (Brass and Coale 1968), have a long history in the field of
demography and are often used to identify and track mortality trends for countries with
poor registration systems. Rather than relying on incomplete or erroneous vital statistics
records, this method uses information that can be easily obtained (women’s age, number
of children born, and number of children surviving) to calculate the proportion of
children who have died to women in certain age groups which, in turn, can be converted
into an estimate of the probability of mortality.

The first step is to determine the proportion of children who have died for women
in five-year age groups (15-19, 20-24, 25-29, and 30-34). The proportions are then
multiplied by an adjustment factor to account for age patterns in childbearing in Brazil.
The adjustment factor is derived from measures of parity as well as coefficients derived
from regional model life tables developed by Coale and Demeny (1983) that contain
information on regional patterns in age-specific mortality. For each age group, the
modified proportions are considered to be relatively accurate probabilities of death to
children prior to age \( a \) \( [q(a)] \). For example, child deaths attributed to women between the
ages of 15 and 19 are used to estimate the probability of death for children under the age
of one, while deaths to women between 30 and 34 are used to estimates the probability of
death for children under the age of five, or child mortality.

In this study, we construct indirect estimates of child mortality rates \( [q(5)] \) for
each municipality in Brazil in the year 2000. Although traditionally women between the
ages of 30 and 34 are used to estimate the probability of death among children prior to
age five, we opt to use younger women, between the ages of 20 and 29. Because the
intent of this study is to examine mortality levels in the year 2000, it is preferable to use
younger women as their experience is more heavily influenced by current levels of child
mortality. In order to use women between the ages of 20 and 29, we start by determining
how the proportion of children who have died to women between the ages of 20 and 29
relates to the probability of death to children prior to a specific age. To accomplish this,
we find the \( q(a) \) values for each single year of age using the 2000 Brazil Life Table
(Table 2). We then find the age distribution of children (living and deceased) for women
ages 20 to 29 \( [c_{20-29}(a)] \) from 1996 DHS data (Table 1). By multiplying the two values
together, we calculate the proportion of children born to women aged 20 to 29 who would
have died prior to each age according to the age distribution of these women and the
Brazilian Life Table probabilities of death prior to each age. Finally, by summing all the
proportions of children who would have died for each age group, we obtain the total
proportion of children who would have died among women ages 20 to 29 \( (\Sigma d_{20-29}) \) if they
had the age distribution of children born to women in this age group and the mortality
risks of the 2000 Life Table.

Finally, we then compare the value for \( \Sigma d_{20-29} \), 0.0350, with the original \( q(a) \)
values found in the Brazil Life Table and find that the value of \( \Sigma d_{20-29} \) is most similar to
the value of \( q(3) \), 0.0351. Therefore, the proportion of deaths to children for women aged
20-29 is most comparable to the probability of death to children prior to the age of 3.
However, to be consistent with the most commonly used definition of child mortality
(deaths to children under the age of five), we want to convert this value so that it can be
used as an estimate of deaths to children under the age of five. Again, we return to the
Brazil Life Table, this time for multiple years, and find that there is a consistent
relationship between $q(3)$ and $q(5)$ which is defined with the following regression equation:

$$q(5) \sim -0.0018 + 1.1017q(3)$$  \hfill (1)

Thus, by applying this equation to the proportion of children who have died to women between the ages of 20 and 29, $\hat{q}(3)$, we are able to obtain indirect estimates of child mortality, $\hat{q}(5)$, for each municipality in Brazil for the year 2000.

Although the Census data does include a large number of records, there is still a risk of unstable estimates of child mortality for municipalities with small populations. As stated earlier, almost one-quarter of all Brazilian municipalities have less than 5,000 residents and one-half have less than 10,000. After limiting Brazil Census microdata to women between the ages of 20 and 29, one-quarter of the municipalities have less than 80 women and one-half have less than 150. Thus, there is very little data available for constructing estimates of child mortality for these less-populated municipalities, potentially resulting in unstable estimates. This idea is illustrated in Figure 6, which displays the confidence intervals surrounding the proportion of children who have died to women between 20 and 29 years of age ($\Sigma d_{20-29}$) for the 5 municipalities with the largest population sizes and the 5 municipalities with the smallest population sizes. The confidence intervals for the most populated municipalities are quite small; however, the intervals for the five least populated municipalities are, in comparison, extremely large, reflecting very unstable data.
Bayesian Methods

Not very long ago it would have been virtually impossible to calculate accurate estimates of child mortality for municipalities with small populations in Brazil. However, improvements in computer technology and the development of efficient sampling algorithms advances have made it possible to employ Bayesian statistical methods to address many issues related to small sample sizes and unstable estimates (Lawson 2009: 3). The field of Bayesian Statistics is named for Thomas Bayes, an 18th Century mathematician and Protestant minister, who believed that knowledge of previous events can be used to help determine the probability of an event occurring in the future (Bayes 1763).

Today, almost 250 years later, the field of Bayesian Statistics is still based on Bayes’ original theorem. Rather than relying solely on data, Bayesian methods combine data with additional information in order to create stronger and more stable measures. This additional information, known as priors, is often obtained from some previous (or prior) information already known about the topic. Combining observed data with a prior distribution results in a posterior distribution in which each parameter value is now represented by a distribution of values determined by both the data (D) and the prior distribution (θ):

$$P(\theta \mid D) = \frac{P(\theta \mid D)P \mid \theta}{\int P(\theta \mid D)P \mid \theta d\theta}$$ (2)

Constructing a posterior distribution can be an extremely difficult process that involves complex integrations and is virtually impossible when working with complicated models. However, in the 1990s, a sampling method known as the Monte Carlo Markov Chain
(MCMC) was developed that made it possible to simulate a posterior distribution (Gilks, Richardson, and Spiegelhalter 1996).

When using MCMC to simulate a posterior distribution, a Markov chain consists of a set of states in which each state contains a value for the parameter of interest. The Markov chain is a series of random states wherein each future state is only dependent on the current state and is independent of any past states, known as the Markov property. The chain begins in a starting state (defined by an initial probability distribution) and then moves successively to additional states. After a number of steps, the Markov chain should eventually stabilize so that the value of the parameter in each successive state is determined only by the current state and a probability distribution defined by the combined effect of the data and prior distribution. Once the posterior distribution has been effectively simulated, it is then possible to sample values from the Markov chain which accurately represent the values from the posterior distribution.

There are several different MCMC methods that can be used to simulate samples of the posterior distribution. This study uses the Gibbs sampling method which has been used in Bayesian Statistics for over two decades and is now one of the most commonly used methods. Gibbs sampling consists of assigning starting values for all parameters. In the first iteration, the first parameter is assigned an “updated” value obtained by randomly sampling the conditional probability distribution given the values of all other parameters and the observed data/prior distribution. Next, the second parameter is also assigned a new value sampled from the probability distribution given the new value of the first parameter, the starting values of the other parameters, and the data. This process continues until all parameters have been assigned new values resulting in the completion
of one Markov chain. Then, in the next iteration, the first parameter is given a new value dependent, again, on the probability distribution, the data, and the new values assigned to all the other parameters in the first iteration. This process continues until eventually the chain converges so that the values of all parameters are determined by the combined effect of the observed data and the probability distribution, the prior distribution.

Spatial smoothing

In studies in which data are aggregated to a geographical level, prior distributions are often not derived from previous information but instead derived from the data in neighboring areas. In the field of spatial statistics, it is well established that geographically close areas often share a number of similarities. Thus, the distribution and mean of neighboring areas can be “borrowed” to create a prior distribution that strengthens the data for areas with unstable estimates due to small sample sizes (Lawson 2009). In this study, neighbors are defined as municipalities that are physically connected to one another and are identified using the program GeoDa. Overall, there are a total of 32,836 neighbors in Brazil with an average of 6 neighbors per municipality. The smallest number of neighbors is 1 and the largest number of neighbors is 23.

To use prior distributions obtained from neighboring areas, we employ a hierarchical Bayesian model using the program WinBUGS (Bayesian inference Using Gibbs Sampling). The first level of the model consists of the level of mortality in a region in which $Y$ (the number of child deaths reported by women aged 20 to 29 in each municipality $i$) is modeled using a binomial distribution:

$$Y_i \sim \text{binomial}(p_i, n_i)$$

(3)

A binomial distribution is the preferred distribution when dealing with counts in small populations (Arató, Dryden, and Taylor 2006)
where $p_i$ is the probability of a child dying among women aged 20 to 29 in each municipality $i$ and $n_i$ is the number of children born to women aged 20 to 29 each municipality $i$. The probability of a child dying ($p_i$) is modeled using a log-linear model:

$$\text{logit}(p_i) = \alpha + \beta_i$$  

(4)

where $\alpha$ is an unstructured random effect and $\beta_i$ is a spatially structured random effect.

The second level of the hierarchical Bayesian model is the prior distribution for the spatially structured random effect using a conditional autoregressive (CAR) model:

$$f(\beta | \tau) \propto \exp\left[-\frac{\tau}{2} \sum_i \sum_{i+1} w_{i,i+1} (\beta_i - \beta_{i+1})^2\right]$$  

(5)

which identifies dependence between neighboring areas by making $w_{i,i+1} = 1$ if municipalities $i$ and $i+1$ are neighbors and $w_{i,i+1} = 0$ if they are not. Additionally, the CAR model also includes the hyperparameter $\tau$ which denotes how similar neighboring areas should be. Due to uncertainty in the degree of similarity in neighboring areas, in the third level of the hierarchical model, $\tau$ is assigned its own distribution, a hyperprior distribution, with a very weak gamma distribution:

$$\tau \sim \gamma(0.5, 0.0005)$$  

(6)

Results

Crude municipal-level estimates of child mortality

Crude estimates of child mortality are mapped for each municipality in Figure 7. Of the 5,505 municipalities, $q(5)$ values are less than 0.025 for 28% of the municipalities, between 0.025 and 0.049 for 34%, between 0.050 and 0.099 for 24%, and 0.100 and higher for 4%. Additionally, 10% of municipalities have insufficient data to create estimates of child mortality. In terms of spatial patterns, municipalities with higher
estimates of child mortality appear to be somewhat more concentrated in the northern part of the country while municipalities with lower estimates are concentrated more in the southern areas. However, the spatial pattern is relatively weak as there is great variation in \( q(5) \) values for municipalities in all regions of the country.

Further descriptive information about the crude estimates of child mortality is presented in Table 3. The mean \( q(5) \) value for all municipalities is 0.043 with the Northeast demonstrating the highest value (0.062) and the South demonstrating the lowest (0.031). The variation in \( q(5) \) values for all municipalities is quite large with a range of 0.001 to 0.274, a variance of 0.00086, and a standard deviation of 0.029. Within the different regions, variation is highest for the North and lowest for the Southeast although the difference in standard deviations between the two regions is only 0.008. In addition, 534 municipalities had no reported child deaths among women between the ages of 20 and 29, making it impossible to calculate \( q(5) \) values for these areas.

As stated previously, one of the difficulties in constructing estimates in child mortality for areas with small population sizes is that the estimates tend to be unstable with a few more or a few less deaths greatly impacting estimates. To see how variation in \( q(5) \) values relates to sample size, Figure 8 plots \( q(5) \) values by the number of women aged 20 to 29 sampled in each municipality\(^3\). As expected, variation in \( q(5) \) values is highest in the municipalities with the smallest sample sizes and decreases substantially as

\(^3\) Due to scaling issues, 6 municipalities with very large sample sizes of women (20,000 or more) are omitted from the graphs. This had no impact on measures of variance or standard deviation.
sample size increases. Figure 9, which plots $q(5)$ values for the municipalities with the 10% smallest sample sizes, shows a much greater dispersion of values when compared to Figure 10, which is limited to the municipalities with the 10% largest sample sizes. The range in $q(5)$ values for areas with the smallest sample sizes is between 0.011 and 0.211, compared with 0.012 to 0.068 for the areas with the largest sample sizes. Additionally, compared with Figure 10, the variance and standard deviation of $q(5)$ values in Figure 9 are 92 and 73% higher.

In Brazil, child mortality is highly associated with female education, both on at the level of the individual (e.g. Merrick 1985; Sastry 2004) and the community (e.g. Goldani et al. 2002; Alves 2003). As a result, if municipal-level estimates of child mortality are accurate, areas with lower levels of education should exhibit higher $q(5)$ values while areas with higher levels of education should exhibit lower values.

Figure 11, which plots municipal-level estimates of child mortality by the mean number of years of schooling of women aged 25 and above, does show some evidence of this relationship as child mortality levels tend to decrease as female education increases. In fact, the correlation coefficient of -0.44 is statistically significant. However, it is important to note that there is a high level of variability in estimates of child mortality at all levels of education.

When limiting the data to municipalities with the 10% smallest sample sizes (Figure 12), the relationship between child mortality and education is virtually nonexistent. Not only are the $q(5)$ values widely dispersed across all levels of education
but the correlation coefficient (-0.8) is extremely weak and not statistically significant. In contrast, in Figure 13, which includes only municipalities with the largest sample sizes, the relationship between child mortality and education is strong and consistent with clear visual evidence that higher child mortality in areas with lower levels of female education and vice-versa. Additionally, the correlation coefficient is strong (-0.62) and significant. These findings indicate, once again, that estimates of child mortality in areas with small populations may be very unreliable.

**Municipal-level Bayesian estimates of child mortality**

To attempt to address issues related to sample size and unstable (or missing) estimates of child mortality, I construct Bayesian estimates for each municipality. Figure 14 presents these estimates with the following breakdown: 26% of municipalities had \( \hat{q}(5) \) values of less than 0.025, 48% had values between 0.025 and 0.049, 24% had values between 0.050 and 0.099, and 1% had values of 0.100 and above. In comparison with Figure 4, there is much greater evidence of a spatial pattern in levels of child mortality using a Bayesian approach. Overall, there is far less variation in municipalities that are geographically close and clear and consistent evidence of elevated mortality levels in municipalities in the northern regions of the country and lower levels in municipalities in the southern regions.

Table 4 presents descriptive statistics for municipal-level Bayesian estimates of child mortality. The mean \( \hat{q}(5) \) value for all municipalities is 0.040, a slight decrease from the mean crude value of 0.043 (Table 3). Likewise, the mean \( \hat{q}(5) \) values for each region are all slightly lower using a Bayesian approach however the overall pattern
remains the same as the Northeast continues to have the highest mean value (0.061) and the South has the lowest (0.025). Most importantly, though, is that using a Bayesian approach results in substantial declines in variation in $q(5)$ values, indicating that estimates of child mortality are far more stable using this approach. For the country as a whole, the range in $q(5)$ values decreased from 0.001 to 0.274 for crude estimates to 0.013 to 0.202 for Bayesian estimates. Additionally, the variance decreased 55% from 0.0009 to 0.0004 and the standard deviation decreased 31% from 0.029 to 0.020. Finally, by “borrowing” data from neighboring areas, we can now create estimates of child mortality for areas in which it is not possible using only the indirect estimation approach and, thus, have no municipalities with missing $q(5)$ values.

Figure 15 plots Bayesian estimates by the sample size of women aged 20 to 29 in each municipality. While estimates are more varied for municipalities with small sample sizes of women, the level of variation is smaller when using a Bayesian approach. Compared with the crude estimates (Figure 8), the Bayesian estimates are more clustered around a smaller range of values and, as seen above, have both lower variance and standard deviation.

Much of the improvement in variation occurs within municipalities with small population sizes. Compared with Figure 9 above, in Figure 16 $q(5)$ values are much more concentrated around a smaller range of values and all measures of variation are lower using a Bayesian approach. Whereas previously the $q(5)$ values ranged from 0.011 to 0.211, all values now fall between 0.016 and 0.096. Additionally, variance decreased
by 85% from 0.0013 to 0.0002 and the standard deviation decreased 62% from 0.037 to 0.014.

In contrast, the graph for Bayesian estimates of child mortality for municipalities with the 100 largest sample sizes (Figure 17) is virtually identical to the graph for crude estimates of child mortality (Figure 10). While the range is only slightly different (0.012-0.068 for crude estimates and 0.014-0.068 for Bayesian estimates), both the variances and standard deviations are identical. This is due to the fact that the effect of the prior distribution is much less for areas with bigger sample sizes and, thus, estimates in these areas will be primarily determined by the data even when using a Bayesian approach.

To assess the reliability of the Bayesian estimates of child mortality, Figures 18-20 show Bayesian \( q(5) \) values by the mean number of years of schooling of women aged 25 and above. Using only crude estimates of child mortality (Figures 11-13), the relationship between education and mortality tends to be fairly weak (especially in municipalities with small population sizes). However, after using a Bayesian approach to estimate child mortality, the relationship between these two variables improves substantially. Visually, it is much clearer in Figure 18 than Figure 11 that municipalities with lower education have higher mortality levels and municipalities with higher education have lower mortality levels. In addition, the correlation coefficient is much larger using Bayesian vs. crude estimates of child mortality (-0.63 vs. -0.44).

Again, the greatest improvement in \( q(5) \) values occurs within municipalities with small samples of women. In Figure 19 the relationship between education and child mortality is clear, both visually and via the statistically significant correlation coefficient. In fact, the correlation coefficient (-0.62) for the least populated municipalities does not
differ greatly from that seen in the previous graph for all municipalities. In contrast, in Figure 12 which plotted crude $q(5)$ values by mean female education, there was no significant association and the correlation coefficient value was -0.08. These findings indicate that using a Bayesian approach results in improvement in the reliability in estimates of child mortality. Finally, Figure 20 plots $q(5)$ values by mean female education for the municipalities with the 10% largest samples of women aged 20 to 29. Once again, there is little difference between crude and Bayesian estimates when examining areas with larger sample sizes (correlation coefficients of -0.62 and -0.66, respectively), a result of the fact that the model requires that priors have a very limited effect when data is abundant and stable.

**Discussion**

Although the level of child mortality in Brazil has improved substantially in the last several decades, there has been little progress made in recent years and there is much work left to be done. Those working towards further reductions in child mortality for countries such as Brazil now advocate that to effectively combat child mortality it is essential that efforts be focused more on the local levels. Consequently, the construction of reliable estimates of child mortality for small geographical areas is necessary for these efforts to move forward. The purpose of this study was to present a method which would allow for more reliable and accurate measures of child mortality for all municipalities throughout Brazil. Using a Bayesian approach that “borrows” information from neighboring municipalities, we have demonstrated how estimates of child mortality become far more stable and dependable. It is our hope and goal that these estimates (and this method) can be employed by those working towards improvements in child mortality.
to effectively understand how child mortality varies at the local level, eventually resulting in strategies that will lead towards continued improvements in child mortality rates throughout all areas of the country.
Figure 1. Municipal-level UNDP child mortality rates*: Brazil, 2000.

Source: UNDP 2009
*Deaths to children under the age of 5 per 1,000 live births
Figure 2. Municipal-level UNDP child mortality rates* by population size: Brazil, 2000.

Source: UNDP 2009

*Deaths to children under the age of 5 per 1,000 live births
Figure 3. Municipal-level UNDP child mortality rates* by population size: Municipalities with the 10% smallest population sizes, Brazil, 2000.

Source: UNDP 2009
*Deaths to children under the age of 5 per 1,000 live births
Figure 4. Municipal-level UNDP child mortality rates* by population size: Municipalities with the 10% largest population sizes, Brazil, 2000.

Source: UNDP 2009

*Deaths to children under the age of 5 per 1,000 live births
<table>
<thead>
<tr>
<th></th>
<th>Range</th>
<th>Variance</th>
<th>Standard Deviation</th>
<th>Correlation with education</th>
</tr>
</thead>
<tbody>
<tr>
<td>All municipalities</td>
<td>1.76-312.5</td>
<td>427.04</td>
<td>20.67</td>
<td>-0.26**</td>
</tr>
<tr>
<td>Municipalities with 10%</td>
<td>8.55-312.5</td>
<td>793.75</td>
<td>28.17</td>
<td>-0.04</td>
</tr>
<tr>
<td>smallest populations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Municipalities with 10%</td>
<td>5.00-89.73</td>
<td>95.42</td>
<td>9.77</td>
<td>-0.37**</td>
</tr>
<tr>
<td>largest populations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: UNDP 2009

* p<0.05   ** p<0.01
Figure 5. Municipalities of Brazil.

Source: 2000 Brazilian Census.
Table 2. Proportion of children born to women aged 20-29 who would have died \( (d_{20-29}) \) prior to age \( a \) based on the age distribution of children born to women aged 20 to 29 \( [c_{20-29}(a)] \) and the probability of death to children prior to age \( a \) \( [q(a)] \).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( q(a) )</th>
<th>( c_{20-29}(a) )</th>
<th>( d_{20-29} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0260</td>
<td>0.1059</td>
<td>0.0028</td>
</tr>
<tr>
<td>1</td>
<td>0.0315</td>
<td>0.1154</td>
<td>0.0036</td>
</tr>
<tr>
<td>2</td>
<td>0.0338</td>
<td>0.1071</td>
<td>0.0036</td>
</tr>
<tr>
<td>3</td>
<td>0.0351</td>
<td>0.1136</td>
<td>0.0040</td>
</tr>
<tr>
<td>4</td>
<td>0.0362</td>
<td>0.1030</td>
<td>0.0037</td>
</tr>
<tr>
<td>5</td>
<td>0.0369</td>
<td>0.1002</td>
<td>0.0037</td>
</tr>
<tr>
<td>6</td>
<td>0.0375</td>
<td>0.0834</td>
<td>0.0031</td>
</tr>
<tr>
<td>7</td>
<td>0.0379</td>
<td>0.0787</td>
<td>0.0030</td>
</tr>
<tr>
<td>8</td>
<td>0.0382</td>
<td>0.0619</td>
<td>0.0024</td>
</tr>
<tr>
<td>9</td>
<td>0.0385</td>
<td>0.0552</td>
<td>0.0021</td>
</tr>
<tr>
<td>10</td>
<td>0.0387</td>
<td>0.0335</td>
<td>0.0013</td>
</tr>
<tr>
<td>11</td>
<td>0.0390</td>
<td>0.0229</td>
<td>0.0009</td>
</tr>
<tr>
<td>12</td>
<td>0.0394</td>
<td>0.0093</td>
<td>0.0004</td>
</tr>
<tr>
<td>13</td>
<td>0.0399</td>
<td>0.0059</td>
<td>0.0002</td>
</tr>
<tr>
<td>14</td>
<td>0.0405</td>
<td>0.0024</td>
<td>0.0001</td>
</tr>
<tr>
<td>15</td>
<td>0.0413</td>
<td>0.0014</td>
<td>0.0001</td>
</tr>
<tr>
<td>16</td>
<td>0.0423</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>17</td>
<td>0.0435</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>18</td>
<td>0.0448</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

\[ \Sigma c_{20-29}(a) = 1.00 \quad \Sigma d_{20-29} = 0.0350 \]

Sources: DHS 1996; IBGE
Figure 6. Confidence intervals of the proportion of children dead to women aged 20-29 (Σd20-29): 5 most populated and 5 least populated municipios.

Source: 2000 Brazilian Census.
Figure 7. Municipal-level crude estimates of child mortality: Brazil, 2000.

Source: 2000 Brazilian Census
<table>
<thead>
<tr>
<th>Regions</th>
<th>North (n=432)</th>
<th>Northeast (n=1,746)</th>
<th>Southeast (n=1,485)</th>
<th>South (n=905)</th>
<th>Center West (n=403)</th>
<th>Brazil (n=4,971)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.042</td>
<td>0.062</td>
<td>0.032</td>
<td>0.031</td>
<td>0.034</td>
<td>0.043</td>
</tr>
<tr>
<td>Median</td>
<td>0.037</td>
<td>0.057</td>
<td>0.027</td>
<td>0.024</td>
<td>0.029</td>
<td>0.036</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.001</td>
<td>0.004</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.179</td>
<td>0.255</td>
<td>0.258</td>
<td>0.274</td>
<td>0.167</td>
<td>0.274</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00057</td>
<td>0.00088</td>
<td>0.00049</td>
<td>0.00066</td>
<td>0.00050</td>
<td>0.00086</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.024</td>
<td>0.030</td>
<td>0.022</td>
<td>0.026</td>
<td>0.022</td>
<td>0.029</td>
</tr>
<tr>
<td>Missing</td>
<td>17</td>
<td>40</td>
<td>180</td>
<td>254</td>
<td>43</td>
<td>534</td>
</tr>
</tbody>
</table>

Source: 2000 Brazilian Census
Figure 8. Municipal-level crude estimates of child mortality by sample of women aged 20 to 29: Brazil, 2000.

Source: 2000 Brazilian Census
Figure 9. Municipal-level crude estimates of child mortality by sample of women aged 20 to 29: Municipalities with the 10% smallest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 10. Municipal-level crude estimates of child mortality by sample of women aged 20 to 29: Municipalities with the 10% largest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 11. Municipal-level crude estimates of child mortality by mean years of schooling of women aged 25 and above: Brazil, 2000.

Source: 2000 Brazilian Census
Figure 12. Municipal-level crude estimates of child mortality by mean years of schooling of women aged 25 and above: Municipalities with the 10% smallest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 13. Municipal-level crude estimates of child mortality by mean years of schooling of women aged 25 and above: Municipalities with the 10% largest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 14. Municipal-level Bayesian estimates of child mortality: Brazil, 2000.

Source: 2000 Brazilian Census
Table 4. Descriptive statistics for municipal-level Bayesian estimates of child mortality: Brazil, 2000

<table>
<thead>
<tr>
<th>Regions</th>
<th>North (n=449)</th>
<th>Northeast (n=1,786)</th>
<th>Southeast (n=1,665)</th>
<th>South (n=1,159)</th>
<th>Center West (n=446)</th>
<th>Brazil (n=5,505)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.041</td>
<td>0.061</td>
<td>0.029</td>
<td>0.025</td>
<td>0.031</td>
<td>0.040</td>
</tr>
<tr>
<td>Median</td>
<td>0.039</td>
<td>0.058</td>
<td>0.027</td>
<td>0.024</td>
<td>0.030</td>
<td>0.033</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.019</td>
<td>0.023</td>
<td>0.014</td>
<td>0.013</td>
<td>0.017</td>
<td>0.013</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.100</td>
<td>0.202</td>
<td>0.106</td>
<td>0.078</td>
<td>0.059</td>
<td>0.202</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00012</td>
<td>0.00033</td>
<td>0.00008</td>
<td>0.00004</td>
<td>0.00005</td>
<td>0.00039</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.011</td>
<td>0.018</td>
<td>0.009</td>
<td>0.007</td>
<td>0.007</td>
<td>0.020</td>
</tr>
<tr>
<td>Missing</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: 2000 Brazilian Census
Figure 15. Municipal-level Bayesian estimates of child mortality by sample of women aged 20 to 29: Brazil, 2000.

Source: 2000 Brazilian Census
Figure 16. Municipal-level Bayesian estimates of child mortality by sample of women aged 20 to 29: Municipalities with the 10% smallest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 17. Municipal-level Bayesian estimates of child mortality by sample of women aged 20 to 29: Municipalities with the 10% largest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 18. Municipal-level Bayesian estimates of child mortality by mean years of schooling of women aged 25 and above: Brazil, 2000.

Source: 2000 Brazilian Census
Figure 19. Municipal-level Bayesian estimates of child mortality by mean years of schooling of women aged 25 and above: Municipalities with the 10% smallest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
Figure 20. Municipal-level Bayesian estimates of child mortality by mean years of schooling of women aged 25 and above: Municipalities with the 10% largest sample sizes, Brazil, 2000.

Source: 2000 Brazilian Census
References


